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RARE EVENTS QUEUEING SYSTEM – REQS

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Abstract: *The paper deals with the queueing system for customers with Poisson's arrival process with the intensity λ and two service modes: in the regular service regime of the intensity control μ, customers are served with a probability of $p\approx1$, and in the special service regime provided to special customers, they are served with the intensity ξ. Special customers access REQS with a complementary probability of $(1-p)\approx0$. A special customer service is analogous to a rare event. The standard methodology has developed analytical patterns for stationary REQS with one service channel and an infinite number of positions in the queue. The analysis of the work of REQS indicates that, when favorable metering parameters $\rho=\lambda/\mu>2$ are concerned, the queueing system is resistant to collapse when an occurrence comes up. However, the regular time losses of regular customers in REQS are extremely high. For this reason, this is the first time that the system stabilization period is being promoted, representing the time interval for the completion of a special customer service before REQS. The analytical apparatus of the queueing system has shown an excellent adaptability to the heterogeneous demands for services μ and special customers, with a low service intensity ξ where $\mu>\xi$. The system can be applied to checkpoint calculations, traffic cuts due to accidents, incidents in industrial systems, i.e. in the case of the occurrence of rare events happening due to anthropogenic and technical factors in the intervals ranging from 10^{-4} to 10^{-6}. The model is not intended for natural hazards.*

***Key words**: collapse, special service, critical probability, stabilization time*

1. Introduction

The development of rare events theory began in the 1970s and was above all aimed at predicting natural hazards (earthquakes) (Cornell, 1968). After the quickly

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obtained results, the importance of the new theory and the application possibilities for the calculation of the hazards induced by anthropogenic factors (e.g. by terrorism) and industrial hazards increased. Since then, rare events theory has become a research field intended to improve the reliability and security of the system (Der Kiureghian, & Liu, 1986; Yang et al., 2015).

From the point of view of systems theory, rare events are characterized by a low frequency of the implementation of the usually uncovered range. The unpredictable and undesirable jargon recognizes rare events caused by the system's current operating regimes as a "black Swan" or a "gray Swan".

The low frequency of rare events makes it impossible to form the necessary statistical set (large datasets) for the significant verification of the distribution of their occurrences. Therefore, it is common to assign an exponential distribution to the distribution of rare events (Zweimuller, 2018; Garnier and Moral, 2006; Jacquemart & Morio, 2016; Ruijters et al., 2019). Such an approach is theoretically justified because of the memoryless properties of the exponential distribution, which completely eliminates the functional relationships between consecutive rare events. Due to the unpredictability and the low probability of their occurrence, the simulation of rare events is a specific analytical task (Morio et al., 2014; Au & Patelli, 2016; Agarwal & De Marco, 2018).

In order to investigate the extreme working conditions caused by the realization of rare events, there are standards in technical systems that, under a rare event, adopt a frequency within the interval ranging from 10^{-4} to 10^{-6} during the lifetime of the system, or as low as 10^{-8}, during the one hour of the operation of the system (Paté-Cornell, 1994).

In this paper, the single-channel REQS model analyzed is the Markovian, which implies the exponential structure of each parameter. Rare events are substituted with a customer with a specific request, who accesses REQS and who is likely to be a rare event $(1-p)$. A special customer requires to be described by the crucial parameter – the time of the special customer service incomparably greater than the time of the regular customer service. Basically, REQS is a heterogeneous system. The analytical apparatus of the queueing system in the stationary mode of operation shows an exceptional adaptability to the introduction of a special customer. Thanks to analytical elasticity, the REQS limitation modes are easily calculated, and the regular capacity of the system and the special customer service regions prevent the system from collapsing.

2. The Birth-Death Process in REQS. Single-Channel REQS

Allow us now to consider the birth-death process in a system with the homogeneous birth process of the intensity λ. Let the dying process be heterogeneous, with the standard mortality intensity μ of the probability of $p\approx1$. The mandatory working condition is $\lambda<\mu$, with the complementary probability of $(1-p)\approx0$, which represents the special dying process occurring after the regular dying process. The intensity of the special dying process has the intensity of $\xi\neq\mu$, where $\mu>\xi$. Keeping this in mind, the mean death time represents a special case of the complementary probability of $(1-p)\approx0$, which is incomparably longer than the regular one. The graph of the elementary states of this process is presented in Figure 1.

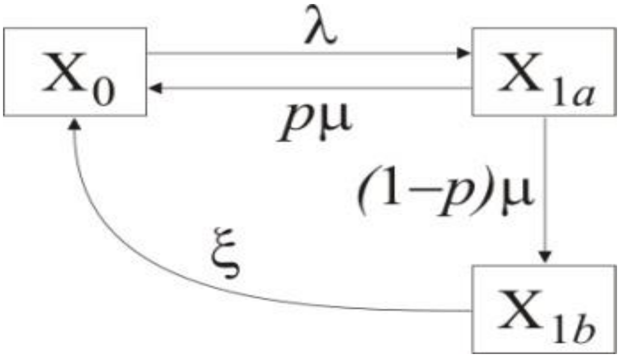

Figure 1. The elementary states of the single-channel REQS

The average dying time is equal to, as in Equation (1):

$$p\int_0^\infty te^{-\mu t}dt+(1-p)\int_0^\infty te^{-\mu t-\xi t}dt=\frac{p}{\mu}+\frac{(1-p)}{\mu}+\frac{(1-p)}{\xi}=\frac{1}{\mu}+\frac{(1-p)}{\xi} \tag{1}$$

The implications of this process for queueing systems are considered under the conditions of the limited value of the probability of the distribution of customers p in the case when the $p\to1$ system is reduced to the standards of the Markovian system with one channel of services(i.e. in a system without customers with special requests), which is expressed by Kendall's notation M(λ)/M(μ)/1/0, Equation (2):

$$\lim_{p\to1}\left(\frac{1}{\mu}+\frac{(1-p)}{\xi}\right)=\frac{1}{\mu} \tag{2}$$

In the case where $p\to0$ (all customers are special customers), the average time of the service described in Raikov's theorems is obtained. The stability of the Poisson stream creates the second boundary result, as in Equation (3). In the boundary conditions of Equation (3), the queueing system is again the standard Markovian queueing system, which is expressed by Kendall's notation M(λ)/M($\mu^{-1}+\xi^{-1}$)/1/0, Equation (3):

$$\lim_{p\to0}\left(\frac{1}{\mu}+\frac{(1-p)}{\xi}\right)=\frac{1}{\mu}+\frac{1}{\xi} \tag{3}$$

The boundary conditions determine the mean time of the services of the single-channel queueing system in the regular operation mode $0<p<1$, Equation (4):

$$\frac{1}{\mu}\le\frac{1}{\mu}+\frac{(1-p)}{\xi}\le\frac{1}{\mu}+\frac{1}{\xi} \tag{4}$$

3. Single-Channel REQS with an Infinite Queue

If the system of the homogeneous birth and heterogeneous dying processes is projected with an infinite number of points in the queue, the reciprocal value obtained from Equation (4) or Equation (5) is the intensity of the customer services in the queue (Figure 2):

$$\mu_q = \frac{\mu\xi}{\xi + \mu(1-p)} \tag{5}$$

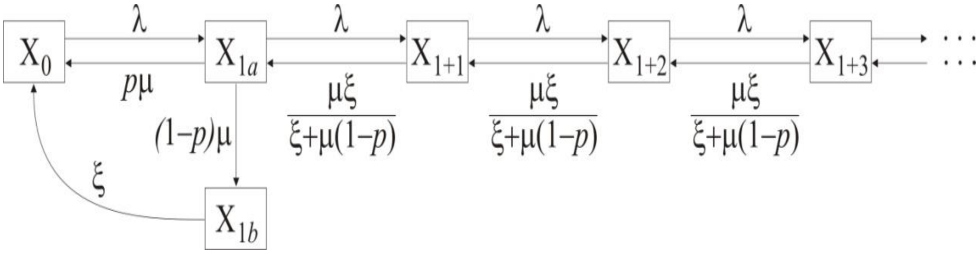

Figure 2. The single-channel REQS with an infinite number in the queue

This system's solution starts from setting balanced equations in the stationary operation mode. The probability of the states X_0, X_{1a}, X_{1b}, X_{1+i} is indicated by the protocol: $P(X_0)=x_0$, $P(X_{1a})=x_{1a}$, $P(X_{1b})=x_{1b}$, and the probability of the state of the queue system for $i\in(1,\infty)$ with $P(X_{1+i})=x_{1+i}$, Equation (6), reads as follows:

$$\begin{aligned}
&X_0: 0 = -\lambda x_0 + p\mu x_{1a} + \xi x_{1b} \Leftrightarrow x_0 = \frac{p\mu x_{1a} + \xi x_{1b}}{\lambda} \\
&X_{1a}: 0 = +\lambda x_0 - (p\mu + (1-p)\mu + \lambda)x_{1a} + \frac{\mu\xi}{\xi + \mu(1-p)} x_{1+1} \\
&X_{1b}: 0 = +(1-p)\mu x_{1a} - \xi x_{1b} \Leftrightarrow x_{1b} = \frac{(1-p)\mu}{\xi} x_{1a}
\end{aligned} \tag{6}$$

In the sequel, all the three probabilities from Equation (6) are shown through the probability of the state x_0, Equation (7):

$$\begin{aligned}
&x_0 = \frac{p\mu x_{1a} + \xi x_{1b}}{\lambda} = \frac{p\mu x_{1a} + \xi \frac{(1-p)\mu}{\xi} x_{1a}}{\lambda} \Leftrightarrow x_{1a} = \frac{\lambda}{\mu} x_0 \\
&x_{1b} = \frac{(1-p)\xi}{\mu} x_{1a} = \frac{(1-p)\xi}{\mu} \cdot \frac{\lambda}{\mu} x_0 \Leftrightarrow x_{1b} = \frac{(1-p)\lambda\xi}{\mu^2} x_0 \\
&0 = +\lambda x_0 - (p\mu + (1-p)\mu + \lambda)x_{1a} + \frac{\mu\xi}{\xi + \mu(1-p)} x_{1+1} \Leftrightarrow \\
&\frac{\lambda^2}{\mu} x_0 = \frac{\mu\xi}{\xi + \mu(1-p)} x_{1+1} \Leftrightarrow x_{1+1} = \frac{\lambda\left(\xi + \mu(1-p)\right)}{\mu\xi} \cdot \frac{\lambda}{\mu} x_0
\end{aligned} \tag{7}$$

Allow us now to proceed with solving the probability in the order of X_{1+i}, Equation (8):

$$
\begin{aligned}
&X_{1+1}: 0 = +\lambda x_{1a} - \frac{\mu\xi}{\xi+\mu(1-p)}x_{1+1} - \lambda x_{1+1} + \frac{\mu\xi}{\xi+\mu(1-p)}x_{1+2} \Leftrightarrow \\
&\lambda\frac{\xi+\mu(1-p)}{\mu\xi}\frac{\lambda^2}{\mu}x_0 = \frac{\mu\xi}{\xi+\mu(1-p)}x_{1+2} \Leftrightarrow x_{1+2} = \left(\frac{\lambda(\xi+\mu(1-p))}{\mu\xi}\right)^2\frac{\lambda}{\mu}x_0 \\
&X_{1+2}: 0 = +\lambda x_{1+1} - \frac{\mu\xi}{\xi+\mu(1-p)}x_{1+2} - \lambda x_{1+2} + \frac{\mu\xi}{\xi+\mu(1-p)}x_{1+3} \Leftrightarrow \\
&x_{1+3} = \left(\frac{\lambda(\xi+\mu(1-p))}{\mu\xi}\right)^3\frac{\lambda}{\mu}x_0; \quad x_{1+4} = \left(\frac{\lambda(\xi+\mu(1-p))}{\mu\xi}\right)^4\frac{\lambda}{\mu}x_0;\ldots
\end{aligned}
\tag{8}
$$

Furthermore, for the probability of all the states in the induction queue, a recurrent form is obtained for the purpose of conducting the probability analysis of the state in the queue of x_{1+i}, Equation (9):

$$
x_{1+i} = \left(\frac{\lambda(\xi+\mu(1-p))}{\mu\xi}\right)^i\frac{\lambda}{\mu}x_0 \tag{9}
$$

The normative condition is as follows, Equation (10):

$$
\begin{aligned}
x_0 + x_{1a} + x_{1b} + \sum_{i=1}^{\infty}x_{1+i} &= x_0 + x_0\frac{\lambda x_0}{p\mu+(1-p)\xi} + x_0\frac{(1-p)\lambda\xi}{p\mu^2+(1-p)\mu\xi} \\
&+ x_0\frac{\lambda}{(p\mu+(1-p)\xi)}\sum_{i=1}^{\infty}\left(\frac{\lambda(\xi+\mu(1-p))}{\mu\xi}\right)^i = 1
\end{aligned}
\tag{10}
$$

Since $\mu >> \xi \wedge 0 \leq p \leq 1$, it follows that the input condition is required of the input intensity λ and the basic intensity of the service $\mu > \lambda$, the sum of Equation (10) being the required geometric order only under the conditions of Equation (11).

$$
\frac{\lambda\xi+\lambda\mu(1-p)}{\mu\xi} < 1 \Leftrightarrow \lambda\xi+\lambda\mu(1-p) < \mu\xi \Leftrightarrow \xi_{min} > \frac{\lambda\mu(1-p)}{\mu-\lambda} \tag{11}
$$

Otherwise, if the condition of Equation (11) is not met, the average time of the ξ^{-1} special customer service is extremely long, and the number of the customers in the queue diverges, i.e. REQS enters into collapse.

With the above-mentioned condition, the value of the geometric order of Equation (10) is equal to that of Equation (12), namely as follows:

$$
\sum_{i=1}^{\infty}\left(\lambda\left(\frac{\xi+\mu(1-p)}{\mu\xi}\right)\right)^i = \frac{1}{1-\lambda\left(\frac{\xi+\mu(1-p)}{\mu\xi}\right)} - 1 = \frac{\lambda\xi+\lambda\mu(1-p)}{\mu\xi-\lambda\xi-\lambda\mu(1-p)} \tag{12}
$$

From the normative condition expressed in Equation (9), the geometric order of Equation (12) gives the probability from x_0, Equation (13):

$$x_0 = \frac{1}{1+\frac{\lambda}{\mu}+\frac{\lambda}{\mu}\cdot\frac{(1-p)\xi}{\mu}+\frac{\lambda}{\mu}\cdot\frac{\lambda\xi+\lambda\mu(1-p)}{\mu\xi-\lambda\xi-\lambda\mu(1-p)}} \quad (13)$$

and all the probabilities of the system from Equations (7) and (9). The average number of the customers in the queue for the fulfilled condition of Equation (11) in the system is equal to that of Equation (14):

$$k_q = \sum_{i=1}^{\infty} i\cdot\left(\lambda\left(\frac{\xi+\mu(1-p)}{\mu\xi}\right)\right)^i = \frac{\left(\frac{\lambda\xi+\lambda\mu(1-p)}{\mu\xi}\right)^2}{1-\frac{\lambda\xi+\lambda\mu(1-p)}{\mu\xi}} = \frac{(\lambda\xi+\lambda\mu(1-p))^2}{\mu\xi(\mu\xi-\lambda\xi-\lambda\mu(1-p))} \quad (14)$$

The average time that the customers spend in the queue is as expressed in Equation (15):

$$t_q = \frac{\bar{k}_q}{\lambda} = \frac{\lambda(\xi+\mu(1-p))^2}{\mu\xi(\mu\xi-\lambda\xi-\lambda\mu(1-p))} \quad (15)$$

4. The Limits of Collapse and Stabilization Time T_{st} in REQS

As in most queueing systems, the basic relationship in Equation (15) determines the operation of the system:

$$\rho = \frac{\lambda}{\mu} \quad (16)$$

For the values of the probability of the findings of special customers "$(1-p)$" and the anticipated average time of special customers ξ^{-1}, the minimum intensity of the regular customer μ_{min} in Equation (17) can be calculated, which guarantees the sustainability of the system:

$$\xi_{min} > \frac{\lambda\mu(1-p)}{\mu-\lambda} \Leftrightarrow \mu_{min} > \frac{\xi_{min}\lambda}{\xi_{min}-\lambda(1-p)} \quad (17)$$

On the contrary, if the conditions from Equation (17) are not satisfied, REQS goes into collapse by diverging the number of the customers in the queue. If the condition for the operation of a single-channel system in Equation (17) is not satisfied, the service intensity may increase (if there is a variable capacity or capacity reserves) or the service additional channels may be introduced into the system.

For the maximum industrial probability of the occurrence of rare events of 10^{-4}, i.e. p=0.9999, the boundary conditions of REQS are presented in Figure 3. The collapse limits are as follows for the different values of ρ:

- $\mu=1.5\lambda \Leftrightarrow \rho=0.75$, $\xi_{min}>0.00030$, or for the relative relation of the intensity of the ordinary and special customer services$\mu/\xi_{min}=5000$, REQS very quickly enters into collapse, and the number of the customers in the queue rapidly diverges. The occurrence of a rare event, i.e. a rare customer with special requests, quickly destabilizes REQS.

- μ=2.0λ⇔ρ=0.50, ξ_{min}>0.00020, or for the relative relation of the intensity of the ordinary and special customer servicesμ/ξ_{min}=10000, REQS relates well to the appearance of a rare customer. The system is hardly introduced into collapse, but such collapse is a consequence of a high burden placed on the regular customers forming a queue, its slow customer service in the queue, and the big losses of time on the part of the customers in the queue.
- μ=3.0λ⇔ρ=0.33, ξ_{min}>0.000150, or for the relative relation of the intensity of ordinary and special customer services μ/ξ_{min}=20000, REQS is hardly introduced into collapse, remains stable for a long time with the low accumulation of the customers in the queue.
- μ=4.0λ⇔ρ=0.25, ξ_{min}>0.000133, or for the relative relation of the intensity of ordinary and special customer services μ/ξ_{min}=30000, REQS behaves similarly to the previous case.

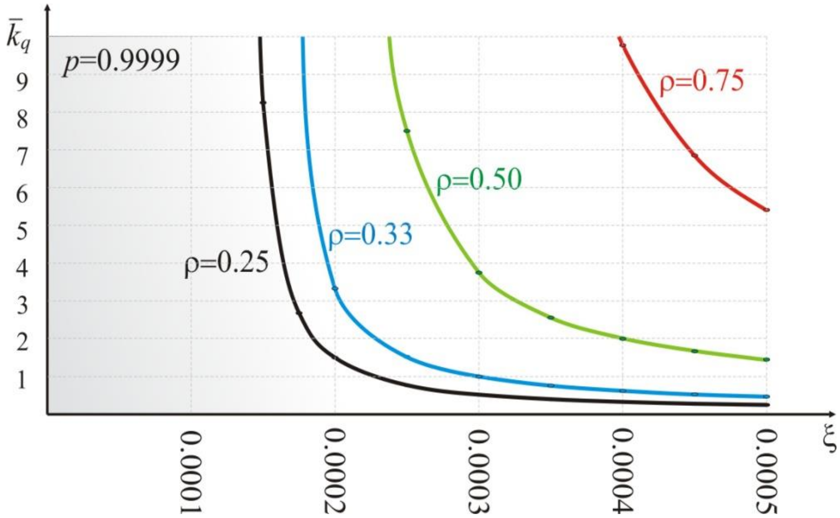

Figure 3. The average number of the customers in the queue for p=0.9999 and the different parameter values of ρ and ξ

If REQS provides special customer service protocols, it does not have to be specifically tailored to rare events. REQS points out the optimization issue regarding the relationship boundary, namely as follows in Equation (18):

$$\frac{\mu}{\xi_{min}} \geq \frac{1}{1-p} \tag{18}$$

which theoretically results in a relative relation μ=2.0λ, i.e. ρ=0.50. One-channel REQS can be optimized. If the inverse value of the parameter of the service ρ^{-1} is accepted for the independent variable (i.e. how many times the intensity of the regular customer service μ is greater than the intensity of incoming customers λ), and if the product $\rho(\xi_{min})^{-1}$ (i.e. the relative parameter of the engagement of the regular and special customer service) is accepted for the dependent variable, then the function shown in Figure 4 is obtained.

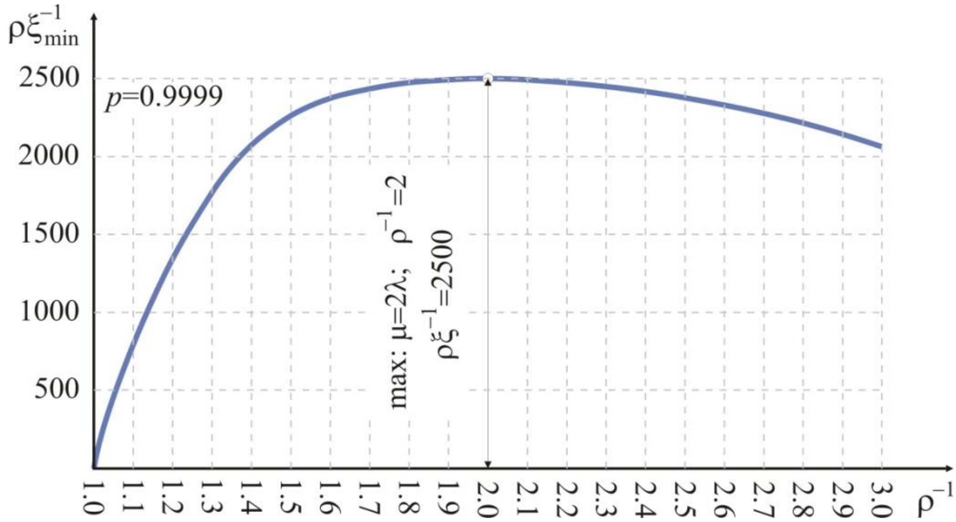

Figure 4. The boundary of the collapse of REQS for the maximum engagement of the service channel

For p=0.9999, the maximum ratio $\rho\xi^{-1}=f(\rho^{-1})$ is obtained for $\mu=2\lambda$. In these circumstances, the maximum time of the special customer service $(\xi_{min})^{-1}$ is equivalent to the time required for the arrival of 5000 regular customers, i.e. the services of 2500 regular customers. For ξ_{min}=0.00020 as per Equation (17), the system is on the edge of collapse.

If the customer service time is reduced by 25%, i.e. if the intensity of special customer services increases to ξ=0.00025 (which is an equivalent to the arrival time of 4000 regular customers, or the service of 2000 customers), the average number of the customers in the queue during the lifecycle of the system without special customers is k_q=7.50. Comparatively, for the queueing system without special customers $M(\lambda)/M(\mu)/1/\infty$, at a ratio $\mu=2\lambda$, the average number of the customers in the system is k_q=0.5, i.e. 15 times smaller than in REQS. It is possible to conclude that the relations $\mu=2.0\lambda$ the REQS are resistant to collapse, but the average number of the customers in the queue during the lifecycle of the system, i.e. the resulting time losses due to the appearance of special customers with the probability of p=0.9999, is/are extremely high.

If the intensity of the services increases to $\mu=3\lambda$, with the same intensity of special customer services ξ=0.00025, the average number of the customers in the queue is 1.527, and for $\mu=4\lambda$ at ξ=0.00025, the average number of the customers in the queue is 0.773.

A standard example of the application of REQS is a survey of traffic accidents. The basis of the numerical example lies in the calculation of the road capacity (Bogdanović et al., 2013) and the application of the queueing system in the calculation of the road capacity (Tanackov et al., 2019). For the mean intensity of the traffic flow of the main roads in the peak period of λ=900 vehicle/h and the maximum throughput of the traffic lane of $\mu\approx$2200 vehicle/h, a special customer service is analogous to the closing of the traffic lane in order to protect the injured, perform surveys and undertake the other operations necessary for the remediation of the accident. A special customer service (i.e. the service of such a customer as a participant in a traffic accident) lasts incomparably longer than the regular customer service. For the likelihood of the occurrence of regular customers from p=0.99995, i.e. the occurrence of an accident on every 20000 vehicles, REQS collapses if the closing time of the traffic lane (the special customer service) is greater than $(\xi_{min})^{-1} \geq$ 13.13h.

However, in the conditions of urban peak periods with twice the intensity of λ=1800 vehicle/h, the collapse limit is the closing of the traffic lane of $(\xi_{min})^{-1} \geq 2.02$h, i.e. for a traffic flow twice as intense, under the same conditions of the regular customer service, the time to collapse is 6.5 times lesser.

Except for the collapse limit of REQS, another important parameter not evaluated in the literature until now is the stabilization time of REQS, which is marked with the tag T_{st}. The users of REQS subjectively and usually negatively react to a loss of the service quality over time T_{st}.

During special customer services, there is an intensive accumulation of regular customers equal to the product of the input stream and the average time of a special customer service, i.e. $\lambda\xi^{-1}$. At the end of the accumulation of regular customers, the regular operation of the system begins with the intensity μ to service to accumulated clusters $\lambda\xi^{-1}$ and new regular customers, who arrive with the intensity λ. Therefore, the difference expressed in Equation (19) must be greater than ρ, i.e. the regular regime of REQS:

$$\lambda\xi^{-1} - (\mu - \lambda)T_{st} \leq \rho \Leftrightarrow T_{st} \geq \frac{\lambda\xi^{-1} - \rho}{(\mu - \lambda)} \qquad (19)$$

For the average time, the special customer services (closing the traffic lane) from ξ^{-1}=2h in the first numerical example (λ=900 vehicle/h) of the system stabilization time are equal to T'_{st}=1.285 h, whereas in the second (λ=1800 vehicle/h) $T''_{st}=8.997\text{h} \approx 9\text{h}$. The vehicle total cumulative time losses in the second numerical case (for the system stabilization period $T''_{st} \approx 9$h) are equal to 84000 vehicle·h, or 3500 vehicle·days. In the first numerical example, the time losses are about 7.5 times smaller.

For a well-designed intensity, REQS resistance to collapse is certain. However, the appearance of the first "strike" of rare events and the stabilization period T_{st} are a risky REQS time interval. If another special customer appears in the stabilization period, the risks of the collapse of REQS are incomparably larger. If t_{cr} is indicated as the critical time elapsed since the beginning of the stabilization period $t_{cr} \in (0, T_{st})$, the critical probability P_{cr} of the occurrence of special-customer rare events in the period passed since the beginning of the stabilization is equal to that of Equation (20). Although this probability is lesser than the probability of the appearance of the first special customer, it should not be neglected.

$$P_{cr} = (1-p) \int_{T_{st}-t_{cr}}^{T_{st}} \lambda e^{-\lambda t} dt \leq (1-p) \qquad (20)$$

In addition to the specified case in road traffic, REQS can analogously be applied to the disruptions of the schedule of railways caused by accidents, in river traffic in the case of the malfunctioning of ship locks, in the case of the suspension of air traffic due to bad weather conditions, etc.

REQS can also be applied in indirect cases, without the arrival of special customers. For example, in all systems that serve customers through the application of information systems, a "failure" of the information system can be considered as a phenomenon of a rare event with the probability of (1–p), and the system "rebooting" time can be considered as the intensity of special customer services.

The principle to be followed refers to the classification of the system states that can be either stable (the regular mode), or metastable, or unstable. The arrival of customers with special requests always introduces the queueing system into a metastable state, and the appearance of customers with special requests at a critical time $t_{cr} \in (0, T_{st})$ introduces the system into an unstable state.

5. Conclusion

REQS modeling and analyzing indicate that the resistance of the system to the occurrence of rare events (special customers) is based on the capacity of the regular operation mode. If the intensity of the services μ in the conditions of the usual occurrences of rare events from 10^{-4} to 10^{-6}, and when a special customer service lasts incomparably longer than a regular customer service, namely several thousand times (up to 10,000 times) as long, for the relative relationships of $\mu \geq 2\lambda$, the boundary collapse of REQS are "so far". The quantity of services can be maintained even in the conditions of disorder. This fact is encouraging for REQS managers.

However, for regular users of REQS, the collapse limit, i.e. the system's capacity, is not the primary parameter. In the implementation of rare events, REQS regularly operates in the destabilization mode. The new parameter of queueing theory, the stabilization time of the T_{st} system, is the key parameter of the quality of the service that special customers (rare events) degrade primarily through regular customers' intensive cumulative time losses. Therefore, the REQS modes can be justified in exceptional, imperative, and most often unwelcome cases.

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