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A NEW FUZZY GARCH MODEL TO FORECAST STOCK MARKET TECHNICAL ANALYSIS

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Abstract: *Decision making process in stock trading is a complex one. Stock market is a key factor of monetary markets and signs of economic growth. In some circumstances, traditional forecasting methods cannot contract with determining and sometimes data consist of uncertain and imprecise properties which are not handled by quantitative models. In order to achieve the main objective, accuracy and efficiency of time series forecasting, we move towards the fuzzy time series modeling. Fuzzy time series is different from other time series as it is represented in linguistics values rather than a numeric value. The Fuzzy set theory includes many types of membership functions. In this study, we will utilize the Fuzzy approach and trapezoidal membership function to develop the fuzzy generalized auto regression conditional heteroscedasticity (FGARCH) model by using the fuzzy least square techniques to forecasting stock exchange market prices. The experimental results show that the proposed forecasting system can accurately forecast stock prices. The accuracy measures RMSE, MAD, MAPE, MSE, and Theil-U-Statistics have values of 18.17, 15.65, 2.339, 301.998, and 0.003212, respectively, which confirmed that the proposed system is considered to be useful for forecasting the stock index prices, which outperforms conventional GARCH models.*

Key words: *Fuzzy time series, Membership function, trapezoidal fuzzy approach, GARCH model, Forecasting.*

1. Introduction

Forecasting is a significant feature in economics, commerce, various branches of science and marketing. It is a technique that predicts the future behavior of output on the basis of present and past output of yield and past trends. The economy of a nation to a great extent relies on upon capital business sector on upon capital

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business sector, forecasting of stock market and their drifts are important factor in attaining significant gains in financial market. In capital and derivative pricing, investment plans, fund distribution and risk control processes, the accurately computation and prediction of financial- volatility plays a vital role (Franke & Westerhoff, 2011; Haugom, Langeland, Molnár, & Westgaard, 2014; A. Y. Huang, 2011), also fuzzy-Garh models for forecasting financial volatility (Hung, 2011a, 2011b; Maciel, Gomide, & Ballini, 2016). The stock price has deep impact in financial event of the country and large-scale economics approach. However, predicting and forecasting the stocks trading, prices and movement is not an easy task because of the serious impact of full-scale financial variable, including general monetary condition, political interference, financial specialist's decision, sudden and unexpected change in security exchanges. Apart from the statistical models that have been used to understand and forecast variations in the stock market, a lot of attention has also been shifted to the applications of various soft computing application. There are different time series models proposed by the different researchers. Due to appropriateness and efficiency Fuzzy time series models are used in different studies (Bisht, Joshi, & Kumar, 2018; Iqbal & Zhang, 2020; Yu, 2005). Fuzzy set theory, provides an authoritative framework to handle with vague or ambiguous problems and can express linguistic values and human subjective decisions of natural language, (Zadeh, 1965). Fuzzy time series was first presented by (Song & Chissom, 1993, 1994). Furthermore, many fuzzy time series models were developed by researchers using different theories (Chen & Tanuwijaya, 2011; Egrioglu, Bas, Yolcu, & Chen, 2020; Hassan et al., 2020; Iqbal, Zhang, Arif, Hassan, & Ahmad, 2020; Lu, Chen, Pedrycz, Liu, & Yang, 2015; Wang, Lei, Fan, & Wang, 2016; Xiao, Gong, & Zou, 2009). Some analysts developed FTS forecsting models using probabilistic fuzzy set theory and reported significant results (Gupta & Kumar, 2019; W.-J. Huang, Zhang, & Li, 2012). Some fuzzy forecasting models in the environment of intuitionistic fuzzy set theory with equal length intervals are developed by (Abhishekh, Gautam, & Singh, 2018),(Bas, Yolcu, & Egrioglu, 2021) and also some work with unequal length intervals introduced by (Lei, Lei, & Fan, 2016) and (Iqbal & Zhang, 2020). In Addition, a novel method to forecast time series data was introduced by (Soto, Melin, & Castillo, 2018), using ensembles of IT2FNN models with fuzzy integrator optimization. There also some studies in which fuzzy based forecasting techniques are compared with classical models like ARIMA (Iqbal, Zhang, Arif, Wang, & Dicu, 2018). Technical analysis is a tool to predict future stock value developments by analyzing the past succession of stock costs. The generalized autoregressive conditional heteroscedasticity (GARCH) model is one of the famous econometric models used to estimates the volatility in financial market, stock markets. GARCH model is an econometric model, to describe an appropriate approach to estimate the in-monetarist markets volatility in monetarist markets, (Engle, 1982).

GARCH models are useful across an extensive range of applications, also they do have boundaries as this model is only part of a solution. Although these models are usually applied to return series, financial decisions are rarely based solely on expected returns and volatilities. These models are parametric specifications that operate best under relatively stable market conditions. GARCH is explicitly designed to model time-varying conditional variances, Generalized Auto-Regressive Conditional Heteroscedasticity models often failed to capture highly irregular phenomena, including wild market fluctuations (e.g., crashes and subsequent rebounds), and other highly unanticipated events that can lead to significant structural change. GARCH models often fail to fully capture the fat tails distribution observed in asset return series. A fat-tailed distribution is a probability distribution that has the property, along with the other heavy-tailed distributions, that its revelations excess skewness or kurtosis. This comparison is often made relative to the normal distribution, or to the exponential distribution. Heteroscedasticity explains some of the fat tail behavior, but typically not all of it. Fat tail distributions, such as student-t, have been applied in GARCH modeling, but often the choice of distribution is a matter of trial and error. For this purpose, fuzzy model is proposed known as Fuzzy Generalized Auto-Regressive Conditional Heteroscedasticity (FGARCH) model in this paper. Although several fuzzy GARCH models based on different statistical and machine learning approaches are developed, such as (Hung, 2009, 2011a; Popov & Bykhanov, 2005), and (Maciel et al., 2016), but our proposed Fuzzy Generalized Auto-Regressive Conditional Heteroscedasticity (FGARCH) model is the best option because it is useful in investment on assets returns but also operates best under wide market fluctuation.

In this paper, a new fuzzy model is proposed known as Fuzzy Generalized Auto-Regressive Conditional Heteroscedasticity (FGARCH) with fuzzy least square techniques and fuzzy trapezoidal approach. The motivation to use trapezoidal membership function is that it outperforms the different types of membership functions when it comes to develop a fuzzy-model for decision making and applicable to real-world applications. The proposed fuzzy model is the best option because it is useful in investment on assets returns but also operates best under wide market fluctuation. The objectives of the current study are explained as: (i) to estimate the unknown parameter by using the Generalized Auto-Regressive Conditional Heteroscedasticity and forecasting fuzzy models, (ii) to articulate the fuzzy model by using the fuzzy least square technique, (iii) to evaluate the comparison between forecast produced from classical model and proposed fuzzy model and also select the best performance model from them.

The remaining paper comprises in the following stages. First section describes the introduction part. Second section briefly explains the earlier work done by the researchers in classical and fuzzy forecasting model. In third section, briefly described the methodology of the classical econometric model "Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH)" and fuzzy model "Proposed Fuzzy Generalized Auto-Regressive Conditional Heteroscedasticity (FGARCH)"by using fuzzy least square method. This section also comprises concept of limitation in Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH), perceptive to move towards fuzzy model. Fourth section included the results obtained from classical and proposed models with comparing the efficiency of the both models by using different endorsements.

2. Basic Theories

2.1. Fuzzy Set

A fuzzy set $\tilde{Z}$ in the universe of information U can be defined as a set of ordered pairs and it can be represented mathematically as

$$Z^{\sim} = \{(x, \mu Z^{\sim}(x)) | x \in U\} \tag{1}$$

Here $\mu_{\tilde{z}}$(x) is degree of membership of x, which assumes values in the range from 0 to 1, i.e., $\mu_{\tilde{z}}$ (x) ∈ [0,1].

2.2. Trapezoidal membership function

Trapezoidal membership function i**s** described using the following equation

$$T(x; a, b, c, d) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{d-x}{d-c} & c \leq x \leq d \\ 0 & d \leq x \end{cases} \tag{2}$$

Where, **x** represents real value within the universe of discourse. **a, b, c, d** represent a x- coordinates of the four heads of trapezoidal and values should validate the following condition a<b<c<d.

2.3. Fuzzy Time Series

The first time (Zadeh, 1965), proposed the fuzzy set theory, it provides a authoritative framework to handle with vague or ambiguous problems and can express linguistic values and human subjective decisions of natural language. Time series models had failed to consider the application of forecasting theory until fuzzy time-series was defined by (Song & Chissom, 1993, 1994).

3. Proposed Fuzzy-Based Methodology

3.1. Generalized Auto-Regressive Conditional Heteroscedasticity (p,q) Model

The generalized autoregressive conditional heteroscedasticity (GARCH) process is an econometric term proposed in 1982 by Robert F. Engle, an economist. In the year 2003, awarded by the Nobel Memorial Prize for Economics, to propose an approach to econometric model to estimate volatility in monetary markets.

Engle (Engle, 1982) and (Bollerslev, 1986) proposed the Generalized ARCH (p,q) model. The general representation of GARCH (p,q) process (ε_t) is defined as,

$$\varepsilon_t = v_t . \sqrt{h_t}, \tag{3}$$

where v_t: is white noise with $\sigma_v^2 = var(v_t) = 1$ and

$$h_t = \omega_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_i h_{t-1} \tag{4}$$

Where, in Eq. (4), ω_0 is a constant, $\sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2$ shows the Auto-regressive conditional heteroscedasticity term, $\sum_{j=1}^{p} \beta_i h_{t-1}$ shows the generalized autoregressive conditional heteroscedasticity term with the parameters $\alpha_0, \alpha_1, \ldots, \alpha_q$ and $\beta_0, \beta_1, \ldots, \beta_p \geq 0$. If $\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i$, the shocks have a decaying impact of the future volatility (Fryzlewicz, 2007).

The general process for a GARCH model involves three steps. The first is to estimate a best-fitting of Auto-regressive model. The second is to compute Auto-correlations of the error terms. The third step is to test for implication. Two other widely used approaches to estimate and predict the financial volatility are the classic historic volatility method, and the exponentially weighted moving average volatility method.

Heteroscedasticity describes the irregular pattern of variation of an error terms, or variable, in a statistical model. In data where heteroscedasticity present, observations do not confirm to a linear pattern, instead, they tend to clusters. The result is that the conclusions and predictive values drawn from the model will not be reliable. GARCH an econometric model, that can be used to analyze a number of different types of financial data series, for instance, macroeconomic data. Financial institutions classically use this model to estimate the volatility of stock returns, bonds and market indices. They resulting information helps to determine the pricing and as well as supports to judge that, which assets will potentially provide higher returns. Furthermore, it helps to forecast the returns of current investments to support in their asset allocation, hedging, risk management and portfolio optimization decisions.

3.2. Fuzzy Generalized Auto-Regressive Conditional Heteroscedasticity

Proposed fuzzy Generalized Auto-Regressive Conditional Heteroscedasticity Model is given as follows with fuzzy parameters:

$$\tilde{h}_t = \omega_0 + \tilde{\alpha}_1 \varepsilon_{t-1}^2 + \tilde{\alpha}_2 \varepsilon_{t-2}^2 + \ldots + \tilde{\alpha}_q \varepsilon_{t-q}^2 + \tilde{\alpha}_1 h_{t-1} + \tilde{\alpha}_2 h_{t-2} + \ldots + \tilde{\alpha}_p h_{t-p} \quad (5)$$

In the Eq. (5) $\tilde{h}_t$ is the estimated fuzzy variable used as output variable, ($\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_{q+1}, \ldots, \tilde{\alpha}_{p+q}$) are the parameters with *q* term known as fuzzy Auto-correlation parameters and fuzzy parameters with *p* term known as fuzzy partial-Autocorrelation. The impreciseness and conciseness have been tackled by connecting parameters with *p* and *q* order into fuzzy parameters.

3.2.1. Fuzzy Least Square Approach

Fuzzy least square approach is an accumulation form of ordinary least square technique. This technique incorporates of goodness of fit and requires a distance between the fuzzy values estimated by the model and vague data that is really pragmatic. Mathematically it is expressed as:

$$d(\alpha_0, \alpha_1) = \left[\int_0^1 f(\eta) d^2\{(\alpha_0, \alpha_1)\eta\} d\eta\right]^{1/2}$$

where α_0 *and* α_1 are two fuzzy numbers. α_0 is a trapezoidal fuzzy number with four points such as $\alpha_0 = \{a_{0m}, a_{0r}, a_{0u}, a_{0v}\}$ and α_1 is another trapezoidal fuzzy number with four points such that $\alpha_1 = \{a_{1m}, a_{1r}, a_{1u}, a_{1v}\}$, and $f(\eta)$ is the weighting function for determining the distance square between two fuzzy numbers. In both fuzzy numbers α_0 *and* α_1, parameters a_{0m}, a_{1m} represent the left fuzzy points, a_{0r}, a_{1m} represent the left center points, a_{0u}, a_{1m} represent the right center fuzzy points, a_{0v}, a_{1m} represented the right fuzzy points.

3.2.2. Estimation of Parameter of Model by Fuzzy Least Square Approach

The parameter of model is estimated by using the fuzzy least square method to gain a unique solution. The fuzzy least square method is defined by the sum of error distance between observed value $O_t(\eta)$ and estimated output value $E_t(\eta)$. Mathematically sum of error distance is represented as:

$$SSE = \sum_{i=1}^{q} \{d[O_t(\eta), E_t(\eta)]\}^2 \tag{6}$$

Where, in Eq. (6) $d(O_t(\eta), E_t(\eta))$ is mathematically represented as:

$$d(O_t(\eta), E_t(\eta)) = [\int_0^1 f(\eta) d^2((O_t)_\eta, (E_t)_\eta) d\eta]^{\frac{1}{2}} \tag{7}$$

where index t denotes the non-fuzzy series of data used in observed value input $O_t(\eta)$ and estimated output value $E_t(\eta)$.

$$E_t(\eta) = \{f(a_m), f(a_r), f(a_u), f(a_v)\}$$

Now from the equation, each fuzzy parameter converted into a generalized auto-regressive conditional heteroscedasticity in the form of functions given below:

$$f(a_m) = \varepsilon_0 + \sum_{i=1}^{q} a_m \varepsilon_{t-i}^2 + \sum_{j=i}^{p} a_m h_{t-j}, \tag{8}$$

$$f(a_r) = \varepsilon_0 + \sum_{i=1}^{q} a_r \varepsilon_{t-i}^2 + \sum_{j=i}^{p} a_r h_{t-j}, \tag{9}$$

$$f(a_u) = \varepsilon_0 + \sum_{i=1}^{q} a_{u\backslash} \varepsilon_{t-i}^2 + \sum_{j=i}^{p} a_u h_{t-j}, \tag{10}$$

$$f(a_v) = \varepsilon_0 + \sum_{i=1}^{q} a_v \varepsilon_{t-i}^2 + \sum_{j=i}^{p} a_v h_{t-j}, \tag{11}$$

where in the above equations, $f(a_m)$ represent the left point function, $f(a_r)$ represent the left center point function, $f(a_u)$ represent the right center function, and $f(a_v)$ represent the right point function.

Now, estimated output value $E_t(\eta)$ expressed on $E_t(\eta) = [L_0, L_1]$, with α-cut fuzzy interval for trapezoidal number can be represent as:

$$E_t(\eta) = [\{f(a_r) - f(a_m)\}\eta + f(a_m), f(a_v) - \eta\{f(a_v) - f(a_u)\}], \tag{12}$$

Similarly, observed value is $O_t(\eta)$ expressed on $O_t(\eta) = [L_0, L_1]$, such as

$$L_0 = \{f(a_r) - f(a_m)\}\eta + f(a_m), L_1 = f(a_v) - \eta\{f(a_v) - f(a_u)\}, \tag{13}$$

where L_0 represent the lower bound and L_1 represent the higher bound.

Now using the values, the observed and estimated output value is obtained by the sum of square error distance: $SSE = \sum_{i=1}^{q} d[O_t(\eta), E_t(\eta)]^2$, In the above expression, $d[O_t(\eta), E_t(\eta)]$ stands the square distance between the observed and estimated output which is shown as given below:

$$d^2(O_t(\eta), E_t(\eta)) = [L_0 - \{[f(a_r) - f(a_m)]\eta + f(a_m)\}]^2 +$$
$$[L_1 - \{f(a_u) + \eta[f(a_u) - f(a_v)\}]^2$$

Now putting the above expression and weighting function in sum of square error (SSE) equation which is given below:

$$SSE = \sum_{i=1}^{q} \int_0^1 f(\eta)\, [L_0 - \{[f(a_r) - f(a_m)]\eta + f(a_m)\}]^2 \times |\varepsilon_{t-1} - h_{t-j}| +$$
$$[L_1 - \{f(a_u) + \eta[f(a_u) - f(a_v)\}]^2 \times |\varepsilon_{t-1} - h_{t-j}|\, d\eta, \tag{14}$$

Using the equation (12) for finding the partial derivation with respect to a_m, a_r, a_u *and* a_v to the get simplified form of equations given as:

$$\sum_{i=1}^{\rho} 2\int_0^1 \eta(\eta-1)x_{ij} \; \lfloor L_0 - \{f(a_r)-f(a_m)\}\eta + f(a_{m)} \rfloor d\xi = 0 \tag{15}$$

$$\sum_{i=1}^{\rho} 2\int_0^1 \eta^2 x_{ij} \; \lfloor -L_0 + \{f(a_r)-f(a_m)\}\eta + f(a_{m)} \rfloor d\xi = 0, \tag{16}$$

$$\sum_{i=1}^{q} 2\int_0^1 \eta \; x_{ij}[-L_1 + f(a_v) - \eta\{f(a_v)-f(a_u)\}]d\eta = 0, \tag{17}$$

$$\sum_{i=1}^{q} 2\int_0^1 \eta(\eta-1)x_{ij}[L_1 - f(a_v) + \eta\{f(a_v)-f(a_u)\}d\eta = 0 \,, \tag{18}$$

Now by solving the integral of the above equations and putting the values in the equations, encompassing the following equation:

$$a_{m_0}\sum_{i=1}^{q} x_{i0}x_{ij} + a_{m_1}\sum_{i=1}^{q} x_{i1}x_{ij} + \Lambda + a_{m_\eta}\sum_{i=1}^{q} x_{i_\eta}x_{ij}$$

Where x_{i0} =1 and $j = 0,1,2,\Lambda\ p$ after simplification, the standard form of above equation is given as:

$$a_{m_0}\sum_{i=1}^{q} x_{i0}x_{ij} + a_{m_1}\sum_{i=1}^{q} x_{i1}x_{ij} + \Lambda + a_{m_\eta}\sum_{i=1}^{q} x_{i_\eta}x_{ij} = \sum_{i=1}^{q} c_i x_{ij} \,,$$

Now

$$a_r\sum_{i=1}^{q} x_{i0}x_{ij} + a_{r_1}\sum_{i=1}^{q} x_{i1}x_{ij} + \Lambda + a_{m_\eta}\sum_{i=1}^{q} x_{i_\eta}x_{ij}$$

Where x_{i0} =1 and $j = 0,1,2,\Lambda\ p$ after simplification, the standard form of above equation is:

$$a_r\sum_{i=1}^{q} x_{i0}x_{ij} + a_{r_1}\sum_{i=1}^{q} x_{i1}x_{ij} + \Lambda + a_{m_\eta}\sum_{i=1}^{q} x_{i_\eta}x_{ij} = \sum_{i=1}^{q} k_i x_{ij} \,,$$

Now

$$a_{u_0}\sum_{i=1}^{q} x_{i0}x_{ij} + a_{u1}\sum_{i=1}^{q} x_{i1}x_{ij} + \Lambda + a_{u_\eta}\sum_{i=1}^{q} x_{i_\eta}x_{ij}$$

Where x_{i0} =1 and $j = 0,1,2,\Lambda\ p$ after simplification, the standard form of above equation is:

$$a_{u_0}\sum_{i=1}^{q} x_{i0}x_{ij} + a_{u1}\sum_{i=1}^{q} x_{i1}x_{ij} + \Lambda + a_{u_\eta}\sum_{i=1}^{q} x_{i_\eta}x_{ij} = \sum_{i=1}^{q} g_i x_{ij} \,, \tag{19}$$

$$a_{v_0}\sum_{i=1}^{q} x_{i0}x_{ij} + a_{v_1}\sum_{i=1}^{q} x_{i1}x_{ij} + \Lambda + a_{v_\eta}\sum_{i=1}^{q} x_{i_\eta}x_{ij}$$

Where x_{i0} =1 and $j = 0,1,2,\Lambda\ p$ after simplification, the standard form of above equation is:

$$a_{v_0}\sum_{i=1}^{q} x_{i0}x_{ij} + a_{v_1}\sum_{i=1}^{q} x_{i1}x_{ij} + \Lambda + a_{v_\eta}\sum_{i=1}^{q} x_{i_\eta}x_{ij} = \sum_{i=1}^{q} s_i x_{ij} \quad , \tag{20}$$

In these above equations i=1, 2..., q, $c_i, k_i, g_i, and\ s_i$ are the outcomes of integral calculation of the above equations. These simple form of equations (15), (16), (17), (18) are represented in the form of matrix.

$$\left.\begin{aligned} A_m &= C \\ A_r &= K \\ A_u &= G \\ A_v &= S \end{aligned}\right\} \tag{21}$$

In Eq. (21), *m* represents the left point matrix, *r* represents the left center point matrix, *u* represent the right center point matrix, *v* and represent the right center point matrix of trapezoidal membership function. Where *C, K, G* and *S* represent the matrix which are obtained after solving the integral. These matrixes are represented as:

$$m = \{a_{m_0}, a_{m_1}\ \Lambda\ a_{m_n}\}^T \ , \quad C = (\sum_{i=1}^{q} C_{t-1}x_{t-1} + \Lambda\ )^T$$

$$r = \{a_{r_0}, a_{r_1}\ \Lambda\ a_{r_n}\}^T \ , \quad K = (\sum_{i=1}^{q} K_{t-1}x_{t-1} + \Lambda\ )^T$$

$$u = \{a_{u_0}, a_{u_1}\ \Lambda\ a_{u_n}\}^T \ , \quad G = (\sum_{i=1}^{q} G_{t-1}x_{t-1} + \Lambda\ )^T$$

$$v = \{a_{v_0}, a_{v_1}, \Lambda\ a_{v_n}\}^T \ , \quad S = (\sum_{i=1}^{q} S_{t-1}x_{t-1} + \Lambda\ )^T$$

where $A = X^T X$ and $X = \begin{bmatrix} 1 & \cdots & x_{1h} \\ \vdots & \ddots & \vdots \\ 1 & \cdots & x_{qh} \end{bmatrix}$ matrix, X is a data matrix and A is the positive definite with rank = n+1. If matrix $A = X^T X$, then inverse of matrix A can be easily determined. Then equation problem consists of unique solution:

$m = A^{-1}C$, $r = A^{-1}K$, $u = A^{-1}G$, $v = A^{-1}S$

4. Computational Analysis

The stock prices index of Gold as input variable from year 2009 to 2017 is given in Appendix A.

Step 1. In our proposed model, possibility of successes is given into five linguistic terms, each linguistic term is represented by the degree of trapezoidal fuzzy numbers. For example,

Very low interval

$$l_1 = [0.0551 \quad 0.0726 \quad 0.0826 \quad 0.0996]$$

$$l_1 = \begin{cases} 0 & x < 0.0551 \\ \frac{x-0.0551}{0.0726-0.0551} & 0.0551 \le x \le 0.0726 \\ 1 & 0.0726 \le x \le 0.0826 \\ \frac{0.0996-x}{0.0996-0.0826} & 0.0826 \le x \le 0.0996 \\ 0 & x > 0.0996 \end{cases}$$

$$l_1 = [0.0551 - 0.0275(1-\eta) \quad 0.0996 + 0.0275(1-\eta)]$$

Low interval

$$l_2 = [\ 0.0826 \quad 0.0996 \quad 0.1271 \quad 0.1441]$$

$$l_2 = \begin{cases} 0 & x < 0.0826 \\ \frac{x-0.0826}{0.0996-0.0826} & 0.0826 \le x \le 0.996 \\ 1 & 0.0996 \le x \le 0.1271 \\ \frac{0.1441-x}{0.1441-0.1271} & 0.1271 \le x \le 0.1441 \\ 0 & x > 1322.5 \end{cases}$$

$$l_2 = [0.0826 - 0.0275(1-\eta) \quad 0.1441 + 0.0275(1-\eta)]$$

Similarly, other computations are also computed regarding average interval, high interval and very high interval, and further arranged the linguistic variable according to the order as follows

i. $\widetilde{K}_{verylow_n} = l_1 = [0.0551 - 0.0275(1-\eta) \quad 0.0996 + 0.0275(1-\eta)]$

ii. $\widetilde{K}_{low_n} = [0.0826 - 0.0275(1-\eta) \quad 0.1441 + 0.0275(1-\eta)]$

iii. $\widetilde{K}_{average_n} = [0.1271 - 0.0275(1-\eta) \quad 0.1886 + 0.0275(1-\eta)]$

iv. $\widetilde{K}_{high_n} = [0.1716 - 0.0275(1-\eta) \quad 0.2226 + 0.0275(1-\eta)]$

v. $\widetilde{K}_{veryhigh_n} = [0.2056 - 0.0275(1-\eta) \qquad 0.2671 + 0.0275(1-\eta)]$

The above expression (i), (ii), (iii), (iv) and (v) shows very low, low, average, high and very high possibility of success for the nth observations. Each linguistics variables are represented as observed prospective output.

Step 2. Now, used the partial derivation of Eqs. (12), (13), (14) and (15) which is show into a following simplified form of equation given as:

$$\sum_{i=1}^{\rho} 2\int_0^1 \eta(\eta-1)x_{ij} \left\lfloor L_0 - \{f(a_r) - f(a_m)\}\eta + f(a_{m)} \right\rfloor d\xi = 0$$

$$\sum_{i=1}^{p}\left[\frac{1}{2}x_{ij}f(a_m) - x_{ij}L_0\right] = 0 \tag{22}$$

$$\sum_{i=1}^{\rho} 2\int_0^1 \eta^2 x_{ij} \left\lfloor -L_0 + \{f(a_r) - f(a_m)\}\eta + f(a_{m)} \right\rfloor d\xi = 0,$$

$$\sum_{i=1}^{p}\left[4x_{ij}L_0 - 5f(a_r)x_{ij} + f(a_m)x_{ij}\right] = 0 \tag{23}$$

$$\sum_{i=1}^{q} 2\int_0^1 \eta \quad x_{ij}[-L_1 + f(a_v) - \eta\{f(a_v) - f(a_u)\}]d\eta = 0,$$

$$\sum_{i-1}^{p}\left[\frac{2}{3}f(a_u)x_{ij} - L_1x_{ij} + \frac{1}{3}f(a_v)x_{ij}\right] = 0 \tag{24}$$

$$\sum_{i=1}^{q} 2\int_0^1 \eta(\eta-1)x_{ij}[L_1 - f(a_v) + \eta\{f(a_v) - f(a_u)\}d\eta = 0$$

$$\sum_{i=1}^{p}\left[\frac{1}{2}f(a_v)x_{ij} - L_1x_{ij}\right] = 0 \tag{25}$$

Step 3. After solving the integral and putting the values of above equations we get the matrix form of equations shown as:

$$m = A^{-1}C, \qquad r = A^{-1}K, \qquad u = A^{-1}G, \qquad v = A^{-1}S$$

where

$A = X^T X$

and $X = \begin{bmatrix} 1 & \cdots & x_{1_h} \\ \vdots & \ddots & \vdots \\ 1 & \cdots & x_{q_h} \end{bmatrix}$,

Matrix A is the positive definite with rank = n+1. If matrix $A = X^T X$, then inverse of matrix A can be easily determined. Then equation problem consists of unique solution.

$$A=\begin{bmatrix} 0.2032 & -0.0482 & -0.0173 & 0.0737 & 0.0532 & -0.0626 & 0.0613 & 0.0387 \\ -0.0482 & 0.4229 & 0.3145 & -0.1760 & 0.4474 & 0.4718 & -0.2032 & -0.5697 \\ -0.0173 & 0.3145 & 0.3816 & -0.1533 & 0.3684 & 0.3459 & -0.1657 & -0.5291 \\ 0.0737 & -0.1760 & -0.1533 & 0.1481 & -0.1800 & -0.2208 & 0.1043 & 0.2478 \\ 0.0532 & 0.4474 & 0.3684 & -0.1800 & 0.7109 & 0.5123 & -0.2705 & -0.5616 \\ -0.0626 & 0.4718 & 0.3459 & -0.2208 & 0.5123 & 0.7365 & -0.1598 & -0.7122 \\ 0.0613 & -0.2032 & -0.1657 & 0.1043 & -0.2705 & -0.1598 & 0.2270 & 0.1765 \\ 0.0387 & -0.5697 & -0.5291 & 0.2478 & -0.5616 & -0.7122 & 0.1765 & 1.1042 \end{bmatrix}$$

"Very Low" Opportunity of Interval

The possibility of very low interval for the N^{th} observation is written as:

$$\widetilde{K}_{verylow_n} = l_1 = [0.0551 - 0.0275(1-\eta) \;\; 0.0996 + 0.0275(1-\eta)] \qquad (26)$$

Using equation (26), interval of very low possibility for the N^{th} observation is to estimate the observed value at different η values. Here we have selected the η values that lies between [0, 1] from table .After putting the value of η = 0.086 in the equation (26), we have evaluated the value of $L_0 = -0.073$ and $L_1 = 0.0361$

Now following the procedure described in section 3 of this paper, the estimated model of "very low opportunity interval" obtained from observations is shown as:

$\widetilde{h}_t$ = (26.523 32.7861 42.667 103.37) + (25.409 32.014 51.406 101.881) ε_{t-1}^2 + (15.1531 27.179 36.716 82.8758) ε_{t-2}^2 + (28.355 34.023 45.5677 106.771) ε_{t-3}^2 + (22.942 31.841 42.475 123.931) ε_{t-4}^2 + (25.043 29.587 38.879 79.749) h_{t-1} + (30.048 43.198 65.913 131.927) h_{t-2} + (22.945 38.4413 64.478 92.9316) h_{t-3} + h_t

The above equation of model represents the "Very Low" linguistic category case that consist of fuzzy parameter in the form of trapezoidal membership function parameters. Similarly, at other levels, the possibility intervals can be computed by following this procedure.

Step 4. Fuzzy logical relationships

To find the fuzzy logical relationship, fuzzified datasets are arranged according to the years. The fuzzified values are given in table 1.

Table 1. Actual prices of Gold stock index with sets of fuzzy

Years	Stock index Prices (returns*100)	Fuzzy Sets
2010	1.485	F_4
2011	60.39	F_{31}
2012	21.89	F_{12}
2013	44.96	F_{28}
2014	29.064	F_{13}
2015	99.956	F_{55}
2016	69.98	F_{41}
2017	13.88	F_{15}

Fuzzy logical relationships are designed from above fuzzified datasets and are presented in table 2. According to the rule of fuzzification, if the time series observation F(t-1) is fuzzified as F_4 in year 2010 and F(t) as F_{31} in year 2011, then F_4 is mapping into F_{31}. In the same manner sets of fuzzy F_{31} in year 2011 is interrelated to F_{12} in year 2012, sets of fuzzy F_{12} in year 2012 is interrelated to F_{28} in year 2013, sets of fuzzy F_{28} in year 2013 is interrelated to F_{13} in year 2014, sets of fuzzy F_{13} in year 2014 is interrelated to F_{55} in year 2015, sets of fuzzy F_{55} in year 2015 is interrelated to F_{41} in year 2016, sets of fuzzy F_{41} in year 2016 is interrelated to F_{15} in year 2017 so in this way all year fuzzy relationship datasets are formed.

Table 2. Fuzzy logical relationships

Years Relationships	Fuzzy Logical Relationships
2010 → 2011	$F_4 \to F_{31}$
2011 → 2012	$F_{31} \to F_{12}$
2012 → 2013	$F_{12} \to F_{28}$
2013 → 2014	$F_{28} \to F_{13}$
2014 → 2015	$F_{13} \to F_{55}$
2015 → 2016	$F_{55} \to F_{41}$
2016 →2017	$F_{41} \to F_{15}$

Fuzzy logical relationship groups (FLRG's)

Using the table 2, fuzzy logical relationship groups (FLRG's) are formed which are given in the table 3. In group 1, relationship of fuzzy F_4 related to F_{33} is mapping in the same way, the fuzzy relationship of F_{33} is mapping on F_{12}, F_4 is mapping on F_{28} in group 3, F_{28} is mapping on F_{13} in group 4, F_{13} is mapping on F_{55} in group 5, F_{55} is mapping on F_{41} in group 6, F_{41} is mapping on F_{15} in group 7. There is no relationship of fuzzy that consist of more than one set that can be merged into another group. Relationship of fuzzy group are shown in below given table.

Table 3. Fuzzy logical relationship groups

Fuzzy Relationship Groups	Fuzzy set Groups
Group 1	$F_4 \rightarrow F_{31}$
Group 2	$F_{31} \rightarrow F_{12}$
Group 3	$F_{12} \rightarrow F_{28}$
Group 4	$F_{28} \rightarrow F_{13}$
Group 5	$F_{13} \rightarrow F_{55}$
Group 6	$F_{55} \rightarrow F_{41}$
Group 7	$F_{41} \rightarrow F_{15}$

Step 5. Fuzzy Forecasted Prices

Using the methods of fuzzy logical relationship groups by Song (Song & Chissom, 1994), forecasted output is determined. All the relationship groups from the above table consist of case 1 which is stated as that in one to one relationship such as $F_i \rightarrow F_k$ then highest degree occurred in F_k at interval μ_k. Forecasted output of the fuzzy generalized auto-regressive conditional heteroscedasticity model is written in below column of table. In this table, year 2011 is forecasted value using the fuzzified interval midpoint values of 2010. The relationship of fuzzy group of year 2010 is $F_4 \rightarrow F_{31}$, according to fuzzification case 1, the highest degree of F_{31} interval is μ_{31} = [1.489, 38.525], so the forecasted output of the year 2011 is the midpoint of μ_{31} which is equal to 20.005. In the same manner all the forecasted prices are obtained which are given in table 4.

Table 4. Forecasted prices of stock index

Year	Actual prices of index (returns*100)	Forecasted prices of index	Fuzzy Relationship Groups	Midpoints of intervals
2010	1.485		$F_4 \rightarrow F_{31}$	20.005
2011	13.88	20.005	$F_{31} \rightarrow F_{12}$	30.649
2012	21.89	30.649	$F_{12} \rightarrow F_{28}$	37.646
2013	29.064	37.646	$F_{28} \rightarrow F_{13}$	52.064
2014	44.96	52.064	$F_{13} \rightarrow F_{55}$	72.148
2015	60.39	72.148	$F_{55} \rightarrow F_{41}$	99.132
2016	69.98	99.132	$F_{41} \rightarrow F_{15}$	128.569
2017	99.956	128.569		

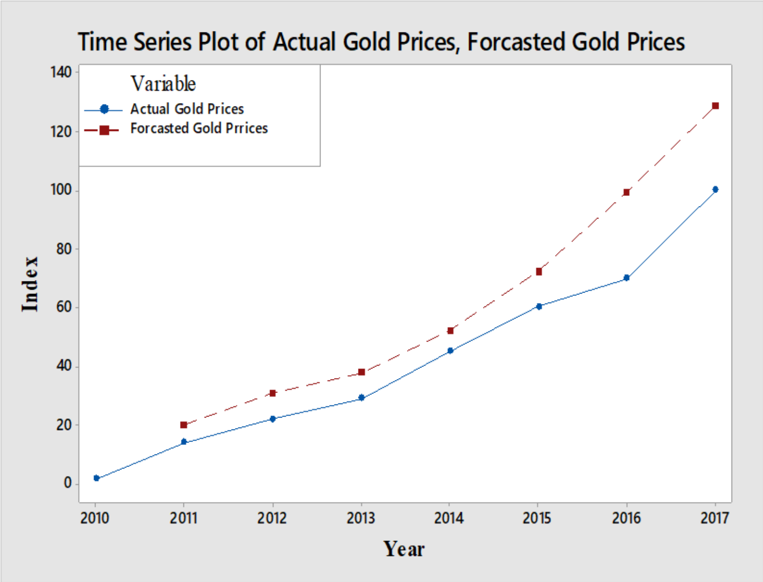

Figure 1. Comparison graph of Actual and Forecasted prices of stock index

In figure.1, it is shown that the actual prices of Gold prices of stock index from year 2010 to 2017 are near to forecasted prices obtained from fuzzy generalized auto-regressive conditional heteroscedasticity from year 2010 to 2017. This graph reflects the regular component which means year to year variation existing in the fuzzy forecasted prices which do not follow any pattern. In actual prices of stock index, from year 2010 to 2013, price pattern shows minor change in trend movement and from year 2013 to 2016 there is a slightly movement and from year 2016 to 2017 there is a slightly downward movement seen in the prices pattern. The forecasted prices pattern shows slightly movement from year 2011 to 2013 and from 2013 to 2015, there is a upward trend seen in prices trend. From year 2016 to 2017, drastic upward movement are seen in forecasted prices.

4.2. Comparison between (GARCH) (p,q) model and (FGARCH) (p,q) model

It is important to know that, which model is performing best and give significant results among classical and proposed fuzzy models. Comparison between GARCH model and proposed FGARCH model is evaluated by using different estimation criteria. Results obtained through GARCH and proposed FGARCH models by using different evaluation methods are given in the table 5.

From above evaluation empirically, criteria's results of proposed Fuzzy Generalized Auto-Regressive Conditional Heteroscedasticity (Fuzzy-GARCH) is smaller and efficient than Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH), which depict that proposed Fuzzy Generalized Auto-Regressive Conditional Heteroscedasticity (Fuzzy-GARCH) perform effectively and efficient as compared to Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH).

Table 5. Different evaluation criteria result obtained from GARCH and Proposed Fuzzy-GARCH

Evaluation criteria	GARCH	Fuzzy-GARCH
Root Mean Square (RMSE)	72.5341	18.170093
Mean Absolute Deviation (MAD)	62.009312	15.6507
Mean Absolute Percentage Error (MAPE)	101.48869	2.339601
Mean Square Error (MSE)	2081.9607	301.998
Theil-U-Statistics	8.53701×10^{-13}	0.003212

To determine whether proposed fuzzy GARCH is appropriate and best model in forecasting than GARCH, we compare the properties of the both classical and proposed model which are given below:

i. Input and output information used in GARCH depend upon a previous function whereas in proposed FGARCH model information are totally based on the fuzzy function.

ii. GARCH model work on the larger observation datasets whereas proposed FGARCH is applicable on small observation as well as larger observations.

iii. GARCH model provides confidence interval whereas proposed FGARCH models give the possibility parameters intervals, which make informal for the forecasted to deal with the possible conditions.

iv. GARCH deals with the conventional fact such as time-fluctuating volatility and volatility crowding, whereas proposed FGARCH deals with forecasting of volatility effect and give more accurate result than classical GARCH.

From above comparison, proposed FGARCH model provides the best forecasted results and best scenario in possibility situation and provide to be effective in spotting the small data outliers.

5. Conclusions

This study is based on basic idea of Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH) in forecasting the prices of stock exchange. A new method based on fuzzy theory is proposed with different mathematical computations and relate this computation in forecasting the stock exchange to determine the efficiency of this model with existing GARCH model. The pragmatic results of the MSE, MFE, MAPE, MAD and RMSE, normalized mean square error (NMSE) of FGARCH model shown in table 4.7 are smaller as compared to GARCH model, which indicates that proposed FGARCH forecasting accuracy is better and perform well than GARCH model. Theil –U-statistic of both models is equal to zero which depicts that Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH) and proposed fuzzy Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH) perfectly forecast the stock prices. The likelihood practice is sensitive to the preliminary value selection and the distribution of data. The presence of uncertainty in data series makes questionable of using the likelihood technique to resolve the uncertainty due to unknown distribution. This limitation is vital in aspects of the forecasting with low accuracy. Future research needs improvement in fuzzy GARCH model regarding estimation of parameters and variability in order to improve the accuracy of model to forecast time series data.

Appendix A: Stock Prices Index of Gold (base 2009-2017)

Date	Gold prices index	Returns of Gold prices	Date	Gold prices index	Returns of Gold prices
12/31/2009	1134.72		1/31/2014	1244.27	0.018292
1/31/2010	1117.96	-0.01499	2/28/2014	1299.58	0.04256
2/28/2010	1095.41	-0.02059	3/31/2014	1336.08	0.027319
3/31/2010	1113.34	0.016105	4/30/2014	1298.45	-0.02898
4/30/2010	1148.69	0.030774	5/31/2014	1288.74	-0.00753
5/31/2010	1205.43	0.04707	6/30/2014	1279.1	-0.00754
6/30/2010	1232.92	0.022297	7/31/2014	1310.59	0.024027
7/31/2010	1192.97	-0.03349	8/31/2014	1295.13	-0.01194
8/31/2010	1215.81	0.018786	9/30/2014	1236.55	-0.04737
9/30/2010	1270.98	0.043407	10/31/2014	1222.49	-0.0115
10/31/2010	1342.02	0.052935	11/30/2014	1175.33	-0.04012
11/30/2010	1369.89	0.020345	12/31/2014	1200.62	0.021064
12/31/2010	1390.55	0.014857	1/31/2015	1250.75	0.04008
1/31/2011	1360.46	-0.02212	2/28/2015	1227.08	-0.01929
2/28/2011	1374.68	0.010344	3/31/2015	1178.63	-0.04111
3/31/2011	1423.26	0.034133	4/30/2015	1198.93	0.016932
4/30/2011	1480.89	0.038916	5/31/2015	1198.63	-0.00025
5/31/2011	1512.58	0.020951	6/30/2015	1181.5	-0.0145
6/30/2011	1529.36	0.010972	7/31/2015	1128.31	-0.04714
7/31/2011	1572.75	0.027589	8/31/2015	1117.93	-0.00929
8/31/2011	1759.01	0.105889	9/30/2015	1124.77	0.006081
9/30/2011	1772.14	0.007409	10/31/2015	1159.25	0.029743
10/31/2011	1666.43	-0.06344	11/30/2015	1086.44	-0.06702
11/30/2011	1739	0.041731	12/31/2015	1075.74	-0.00995
12/31/2011	1639.97	-0.06039	1/31/2016	1097.91	0.020193
1/31/2012	1654.05	0.008512	2/29/2016	1199.5	0.084694
2/29/2012	1744.82	0.052023	3/31/2016	1245.14	0.036655
3/31/2012	1675.95	-0.04109	4/30/2016	1242.26	-0.00232
4/30/2012	1649.2	-0.01622	5/31/2016	1260.95	0.014822
5/31/2012	1589.04	-0.03786	6/30/2016	1276.4	0.012104
6/30/2012	1598.76	0.00608	7/31/2016	1336.66	0.045083
7/31/2012	1594.29	-0.0028	8/31/2016	1340.17	0.002619
8/31/2012	1630.31	0.022094	9/30/2016	1326.61	-0.01022
9/30/2012	1744.81	0.065623	10/31/2016	1266.55	-0.04742
10/31/2012	1746.58	0.001013	11/30/2016	1238.35	-0.02277
11/30/2012	1721.64	-0.01449	12/31/2016	1157.36	-0.06998
12/31/2012	1684.76	-0.02189	1/31/2017	1192.1	0.029142
1/31/2013	1671.85	-0.00772	2/28/2017	1234.2	0.034111
2/28/2013	1627.57	-0.02721	3/31/2017	1231.42	-0.00226
3/31/2013	1593.09	-0.02164	4/30/2017	1266.88	0.02799
4/30/2013	1487.86	-0.07073	5/31/2017	1246.04	-0.01672
5/31/2013	1414.03	-0.05221	6/30/2017	1260.26	0.011283
6/30/2013	1343.35	-0.05261	7/31/2017	1236.84	-0.01894
7/31/2013	1285.52	-0.04499	8/31/2017	1283.04	0.036008
9/30/2013	1348.6	-0.00233	9/30/2017	1314.07	0.023614
10/31/2013	1316.58	-0.02432	10/31/2017	1279.51	-0.02701
11/30/2013	1275.86	-0.03192	11/30/2017	1281.9	0.001864
12/31/2013	1221.51	-0.04449	12/31/2017	1264.45	-0.0138

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